## Many-Body Effect in Bose-Einstein Condensate and Application of Ultracold Quantum Gases

Shohei Watabe<sup>1,\*</sup>

<sup>1</sup>Tokyo University of Science, 1-3 Kagurazaka, Shinjuku-ku, Tokyo, Japan

Bose-Einstein condensate is a ubiquitous phenomenon, which can be seen in the superfluid helium 4, ultracold quantum gases, magnetic insulator, and exciton-polariton systems. In this talk, four topics of the many-body theory of the Bose-Einstein Condensates will be mainly given  $1^{-4}$ . First, I discuss comparative studies on the order of the phase transition and the shift of the critical temperature from that of the ideal Bose gas by the interaction, by using the many-body theory, such as random phase approximation, and many-body T-matrix theory<sup>1</sup>. Second, I propose the many-body theory of BEC for satisfying the Nepomnyashchii-Nepomnyashchii (NN) identity<sup>2</sup>. Our framework satisfies the NN identity for nonzero-temperature BEC, and we found that this approach consistently reproduces the Popov's hydrodynamic result at absolute zero temperature. Third, using the many-body theory for BEC, I discuss and compare the single particle spectral function and the density response function<sup>3</sup>. The coincidence between the single particle excitation and the collective excitation has been believed as a unique nature in BEC, but it has not been explicitly proven by experiments in the superfluid helium 4. I discuss that an ultracold quantum gas is a candidate to experimentally prove this coincidence. Forth, in the ultracold quantum gases, there is not only the single-component scaler BEC, but also BECs with spin degrees of freedom. This spinor BEC shows the  $U(1) \times SO(3)$  symmetry breaking. I show the exact identity, the Hugenholtz-Pines theorem, for the spinor BEC, and the extension of the identity to the case where the external field explicitly breaks the original symmetry<sup>4</sup>. In the end of my talk, I discuss a recent project with AIST for the quantum engineering in utlracold atomic gases.

- <sup>3</sup> S. Watabe, Hidden multiparticle excitation in weakly interacting Bose-Einstein Condensate, Phys. Rev. A 97, 033606 (2018).
- <sup>4</sup> S. Watabe Hugenholtz-Pines theorem with broken  $U(1) \times SO(N)$  or  $U(1) \times SU(N)$  symmetry, arXiv:1709.06319.

<sup>\*</sup> shoheiwatabe@rs.tus.ac.jp

<sup>&</sup>lt;sup>1</sup> S. Watabe, and Y. Ohashi, Comparative studies of many-body corrections to an interacting Bose condensate, Phys. Rev. A **88**, 053633 (2013).

<sup>&</sup>lt;sup>2</sup> S. Watabe, and Y. Ohashi, Green's-function formalism for a condensed Bose gas consistent with infrared-divergent longitudinal susceptibility and Nepomnyashchii-Nepomnyashchii identity, Phys. Rev. A 90, 013603 (2014).